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THE STRESS INTENSITY FACTOR AT THE TIP OF THE CRACK AS INFLUENCED BY RIVET FORCES

Technical Status Report No. 4

For

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
AIR RESEARCH AND DEVELOPMENT COMMAND

By

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CARNEGIE INSTITUTE OF TECHNOLOGY

and

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
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Contract No. AF 49(638)-237

THE EFFECT OF STIFFENERS ON FAST CRACK PROPAGATION
IN THE DESIGN OF FAIL-SAFE AIRCRAFT STRUCTURES

Technical Status Report No. 4
Project No. 17500

TITLE OF REPORT: THE STRESS INTENSITY FACTOR AT
THE TIP OF THE CRACK AS INFLUENCED BY RIVET FORCES

Date: April 16, 1959

James P. Romualdi

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Project Supervisor

THE STRESS INTENSITY FACTOR AT THE TIP OF THE CRACK AS INFLUENCED BY RIVIT FORCES

Introduction

The effect of a riveted stiffener placed perpendicular to and in advance of a crack is twofold: first, the stiffener reduces the buckling of the plate containing the crack and second, rivet forces will be induced such that all rivets in the neighborhood of the crack tip will apply forces to the plate in a direction to oppose further motion of the crack. Both of these effects tend to stop the crack in the vicinity of the stiffener.

In Technical Status Report No. 2, September, 1958, the increase in fracture strength due to reduction of buckling was evaluated. An analytical study of the stress intensity factor, K, and its various ramifications was made in Technical Status Report No. 3, dated January 15, 1959.

In this report experimental data is presented supporting the analytical study made in the previous Status Report. These tests show that it is possible to measure K by use of strain-gages placed near the crack tip. Another test involves determination of K_c, the critical stress-intensity factor for onset of rapid fracture. Later tests investigate the effect of a single rivet force upon stress at the crack tip. Although the results of these tests are here summarized, this testing program is not complete.

The Stress Intensity Factor

In the vicinity of the crack tip, and colinear with the crack, the stress intensity factor is given by(1)

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(1) Raised parentheses refer to references in Bibliography.

$$K = \sqrt{2r}\ \sigma_y \tag{1}$$

This expression is valid for all applied stress patterns, such as uniform tension remote from the crack and splitting forces in the crack.

If the Airy stress function is represented by[2]

$$F = \mathrm{Re}\ \bar{\bar{Z}} + y\ \mathrm{Im}\ \bar{Z} \tag{2}$$

where $\bar{Z}$, Z and Z' are successive derivatives of a function $\bar{\bar{Z}}(z)$, where z is $(x + iy)$, then

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = \mathrm{Re}\ Z - y\ \mathrm{Im}\ Z' \tag{3}$$

and

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = \mathrm{Re}\ Z + y\ \mathrm{Im}\ Z' \tag{4}$$

For the case of a crack perpendicular to a field of uniform tension, σ ,

$$Z(z) = \frac{\sigma z}{\sqrt{z^2 - a^2}} \tag{5}$$

The real part of this function is determined by substitution of the parameters (see Figure 1):

$$\left.\begin{aligned} z &= \rho_1 e^{i\theta_1} \\ z - a &= r e^{i\theta} \\ z + a &= \rho_2 e^{i\theta_2} \end{aligned}\right\} \tag{6}$$

If (6) is substituted into (5), then

$$Z(\rho, \theta) = \frac{\sigma \rho_1}{\sqrt{r \rho_2}}\ \exp i\left[\theta_1 - \frac{\theta + \theta_2}{2}\right] \tag{7}$$

For stresses along a line colinear with the crack, all θ's = 0, $\rho_1 = a + r$ and $\rho_2 = 2a + r$. Expression (7) for this case becomes

$$Z(a,r) = \frac{\sigma(a+r)}{\sqrt{r(2a+r)}} \tag{8}$$

Since y = 0 along the line colinear with the crack, equation (4) becomes

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = \mathrm{Re}\ Z \tag{9}$$

whence

$$\sigma_y = \frac{\sigma(a+r)}{\sqrt{r(2a+r)}} \tag{10}$$

In the region very close to the crack tip, r is much smaller than a, and may be neglected in comparison to it, so that

$$\sigma_y = \frac{\sigma\sqrt{a}}{\sqrt{2r}} \tag{11}$$

If (11) is substituted into (1), then

$$K_\sigma = \sigma\sqrt{a} \tag{12}$$

In a similar manner, the value of K may be determined for the case of splitting forces in the crack, the expression being (see Figure 2)

$$K_p = \frac{2P\sqrt{a}}{\pi\sqrt{a^2 - b^2}} \tag{13}$$

For the case of pinching forces applied near the ends of a crack, (see Figure 3) a pressure distribution is obtained within the crack, and an equal and opposite internal pressure must be applied to make the crack a traction free surface. The K for this case must be integrated

numerically using expression (13) with suitable values for P and b.

K values may be determined experimentally by strain gage measurements near the crack tip. However, a certain error is involved here that is an increasing function of the ratio r/a. For the case of a crack in a uniform stress field, the ratio of K_a, the apparent stress intensity factor to K is calculated by dividing expression (10) by (11)

$$\frac{K_a}{K} = \frac{\frac{\sigma(a+r)}{\sqrt{r(2a+r)}}}{\frac{\sigma\sqrt{a}}{\sqrt{2r}}} = 1 + \frac{3r}{4a} - \frac{5r^2}{32a^2} + \frac{7r^3}{128a^3} \cdots \quad (14)$$

For the case of splitting forces in the crack

$$\frac{K_a}{K} = 1 - \frac{5r}{4a} - \frac{43r^2}{32a^2} - \frac{2b^2r}{a^3} - \frac{177r^3}{128a^3} \quad (15)$$

Experimental Determination of the Stress Intensity Factor

As discussed previously, the stress intensity factor may be determined by measuring strains close to the crack tip. The stress intensity factors, for various crack lengths, were measured in Tests I, II and III. Figure 4 shows the central portion of the test specimens, depicting the crack of length 2a and four strain gages, the centers of which lie at $x = \pm (a + r)$. For Test I, the dimension "a" was one inch. "a" was two inches for Test II and three for Test III. The data from these tests is tabulated in Tables I, II and III and the results plotted on Figures 5, 6 and 7. The theoretical expression for K appears as a straight line on these Figures. Experimental results are in good agreement with the theory.

Measurement of the Critical Stress Intensity Factor

The critical stress intensity factor, K_c, is measured at the onset

of rapid crack propagation. Its value remains relatively constant for a certain range of crack and plate dimensions, providing the properties of the material are constant. For a plate with a centrally located crack,

$$K_c = \sigma_c \sqrt{a_c} \tag{16}$$

where σ_c is the uniform stress applied to the specimen at onset of rapid crack propagation and a_c is the half crack length existing at the same instant.

In Test IV, a centrally notched plate was loaded until failure. A continuous record of the load was made on a recording millivoltmeter. The load was converted to millivolts using a Wheatstone Bridge with one arm consisting of strain gages mounted on a load bar. The millivoltmeter was read at certain loads on the specimen. These readings, along with other test data, are given in Table IV and plotted in Figure 8. The maximum reading of the millivoltmeter was easily determined from the continuous record of load vs time. Figure 8 shows this reading and the corresponding load at fracture.

The crack length at the onset of rapid crack propagation was measured by the extent to which India ink had marked the fracture surface. The ink had been placed in the crack tip at the start of the test. As the crack grew slowly, the ink moved with it, but once the crack started to grow rapidly, the ink was unable to follow.

For a specimen of Alclad 7075-T6, the critical value of the stress intensity factor was observed to be 37,000 lb-(inches)$^{-3/2}$. This value agrees with those of other investigators. Table V[3] gives fracture results for various materials in terms of stress intensity factor and also crack extension force, $\mathcal{G}_c$.

Evaluation of the Effects of Pinching Forces on Stresses in a Plate

Pinching forces are applied to the plate in the region of the crack tip as shown in Figure 9. The forces are applied by two bolts above and below the crack connected by two straps. The elongation of the specimen induces stress in the strap and the pinching force can be determined from the known calibration.

The theoretical pressure distribution[4] between a single pair of opposed forces is shown in Figure 10. Test V is concerned with this pressure distribution. In this test the straps cause pinching forces in absence of a crack. Table VI gives the results of this test. The stresses are measured both with and without straps, the difference between the two stresses being designated $\Delta\sigma_y$. If F is the pinching force in pounds per inch of plate thickness, then $\Delta\sigma_y/F$ should be a constant for any distance r from the crack tip. The average error of four measurements of $\Delta\sigma_y/F$ was 8 percent.

The additional pressure distribution[2] caused by the presence of the crack is shown in Figure 11. This distribution was verified by Test VII. This test was similar in all respects to Test V except that the crack was six inches long and the pairs of pinching forces were six inches apart. Stress readings for this test give the sum of the pressure distributions of Figures 10 and 11. Again, the results of the test are given in terms of $\Delta\sigma_y/F$. The experimental error was 10 percent.

Direct measurement of K values was impossible for Tests V and VII due to the similar magnitudes of the stress fields that were measured. However, the symmetrical field may be measured singly in the absence of the crack and then subtracted from the total stress measured with a crack. The stress intensity factor applies only to the stress field with a discontinuity at the crack tip.

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(2) H. M. Westergaard - "Bearing Pressures and Cracks," Jrnl. App. Mech., June 1939.

(3) J. A. Kies - "The Resistance of Materials to Fracture Propagation by Gunfire Damage," NRL Memo Rpt. No. 594, May 1956.

(4) A. E. H. Love - "A Treatise on the Mathematical Theory of Elasticity," Dover Pub., New York, 1944, pp 209-210.

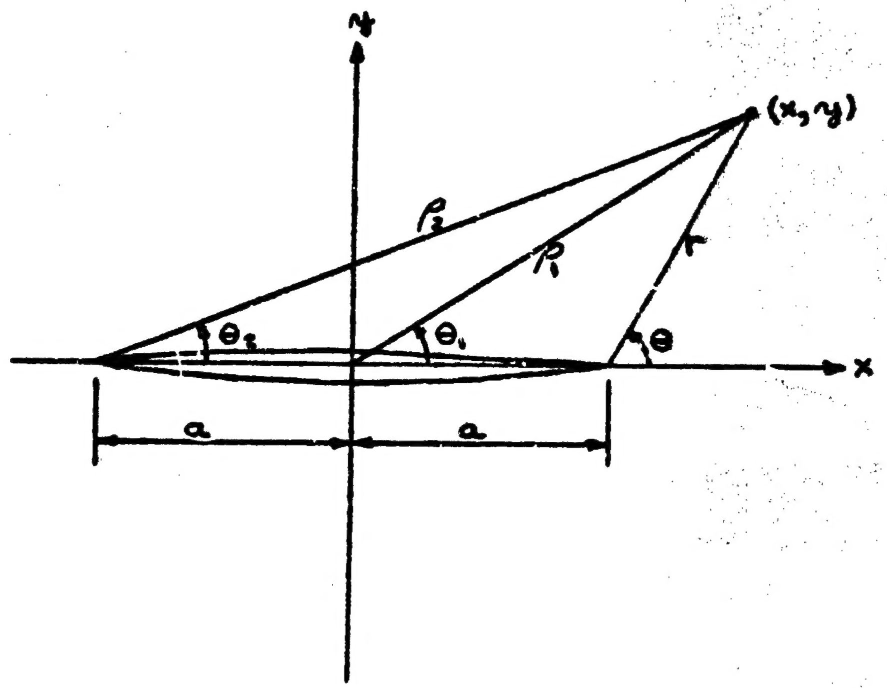


Figure 1 - Configuration of Parameters

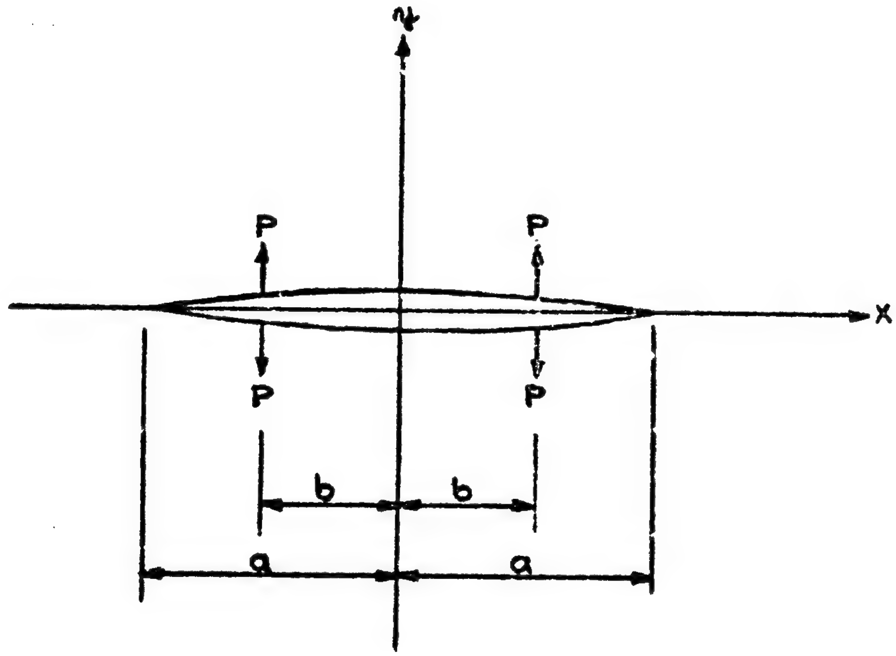


Figure 2 - Splitting Forces in the Crack

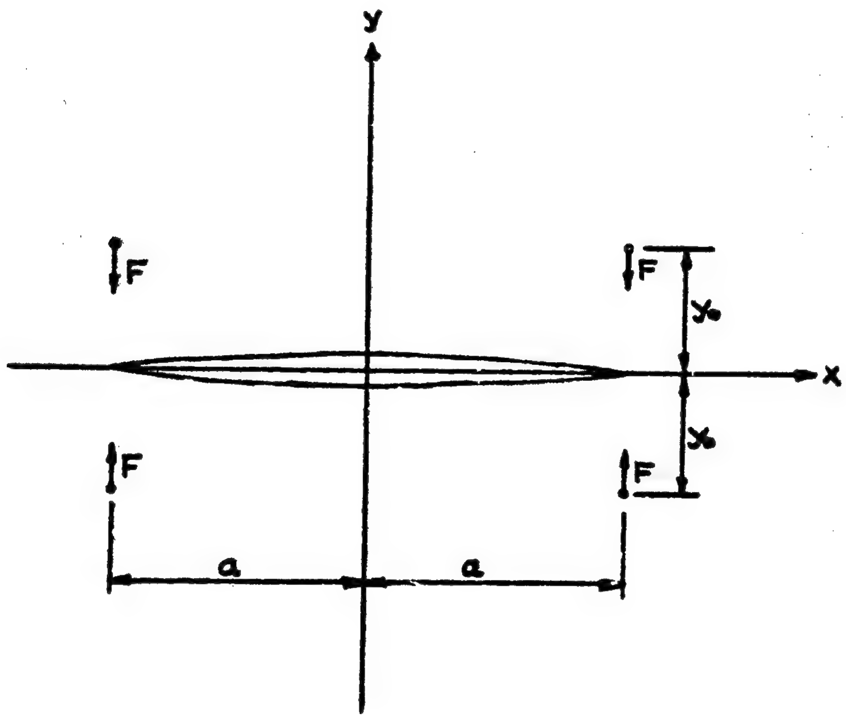


Figure 3 - Configuration of Pinching Forces

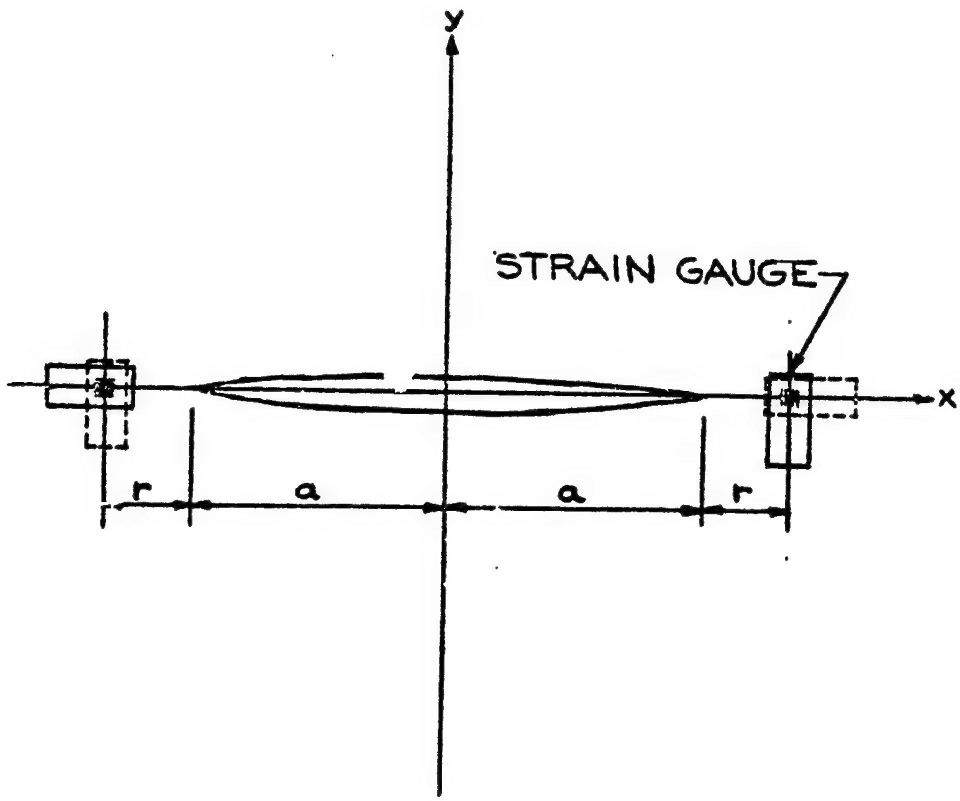


Figure 4 - Crack and Strain Gauges

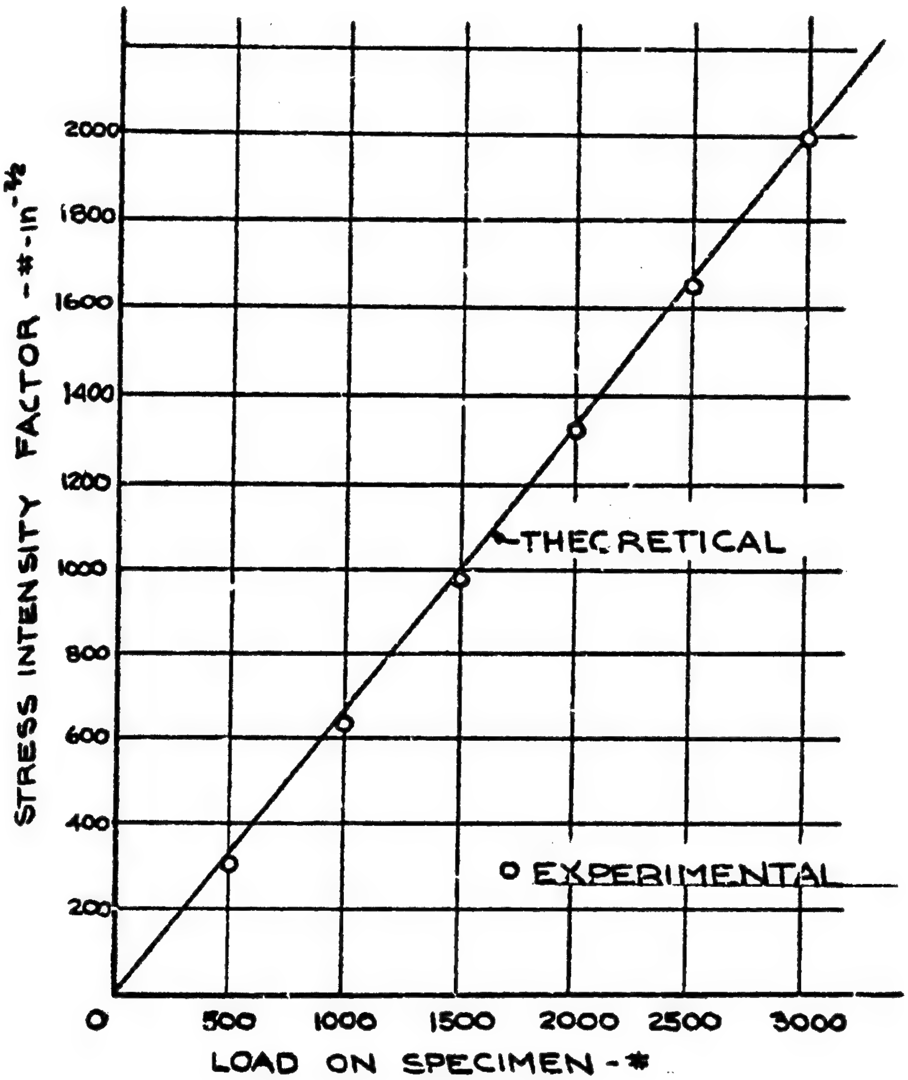


Figure 5 - Thooretical and Experimental Stress Intensity Factors - Test I

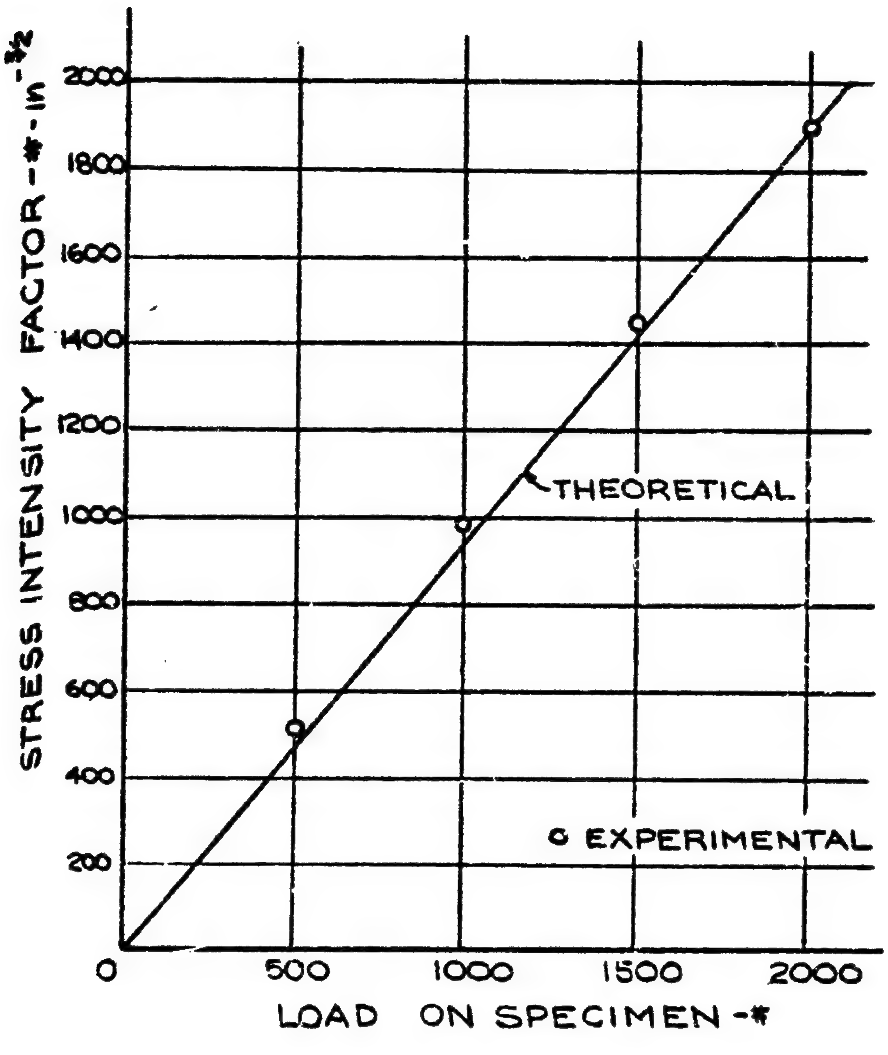


Figure 6 - Theoretical and Experimental Stress Intensity Factors - Test II

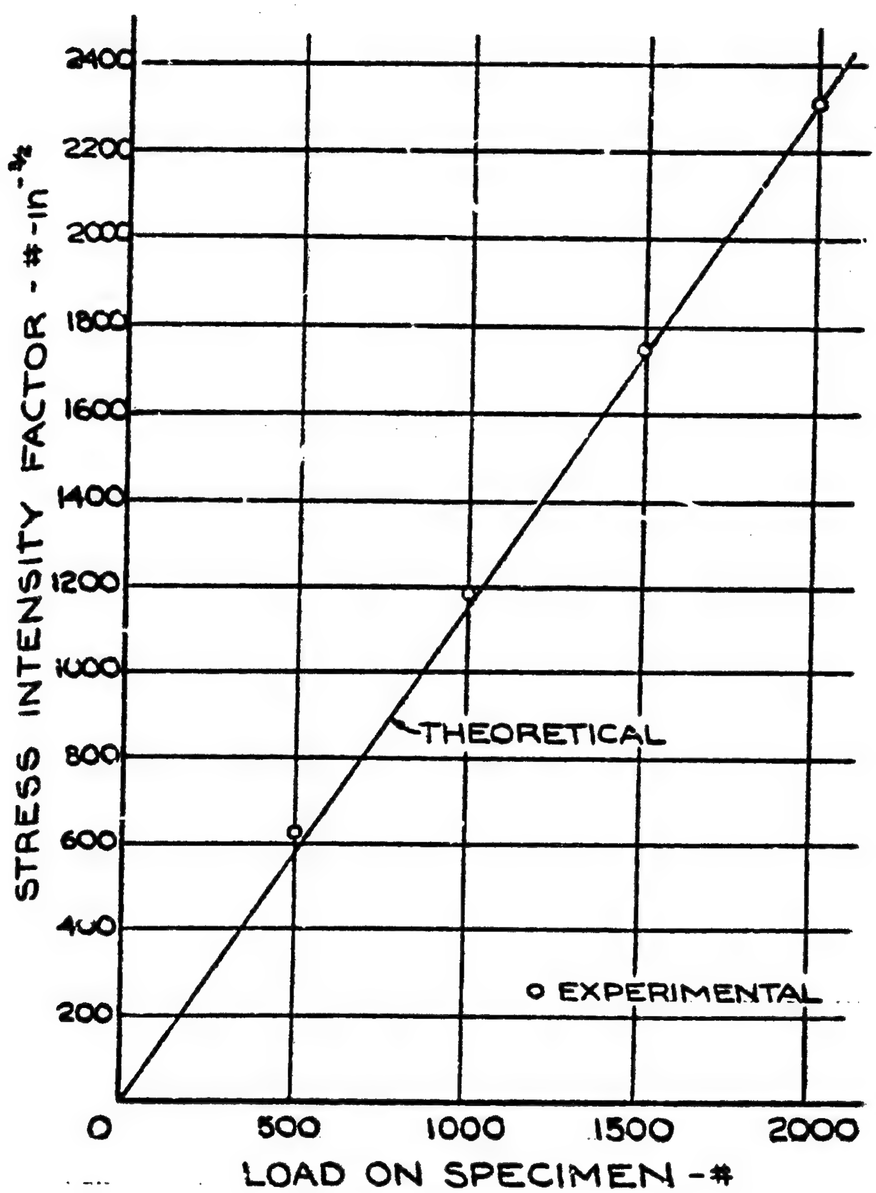


Figure 7 - Theoretical and Experimental Stress Intensity Factors - Test III

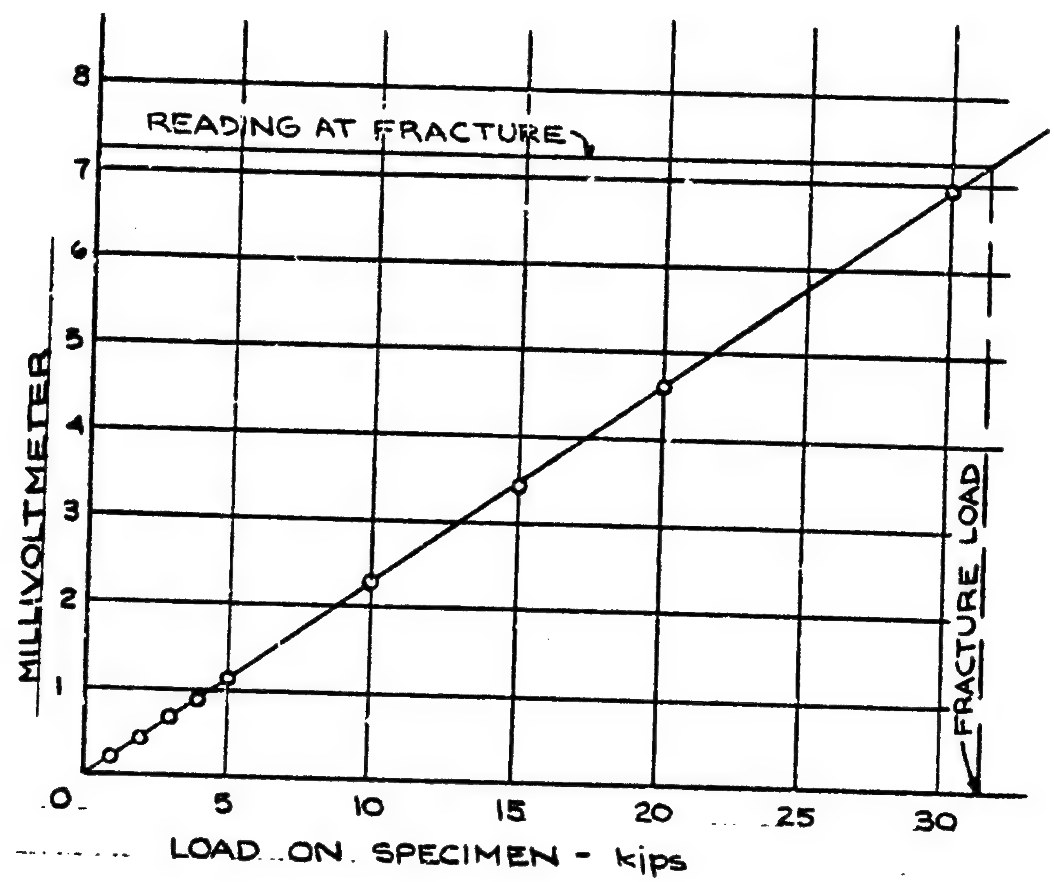


Figure 8 - Millivoltmeter Reading vs. Load for Test IV

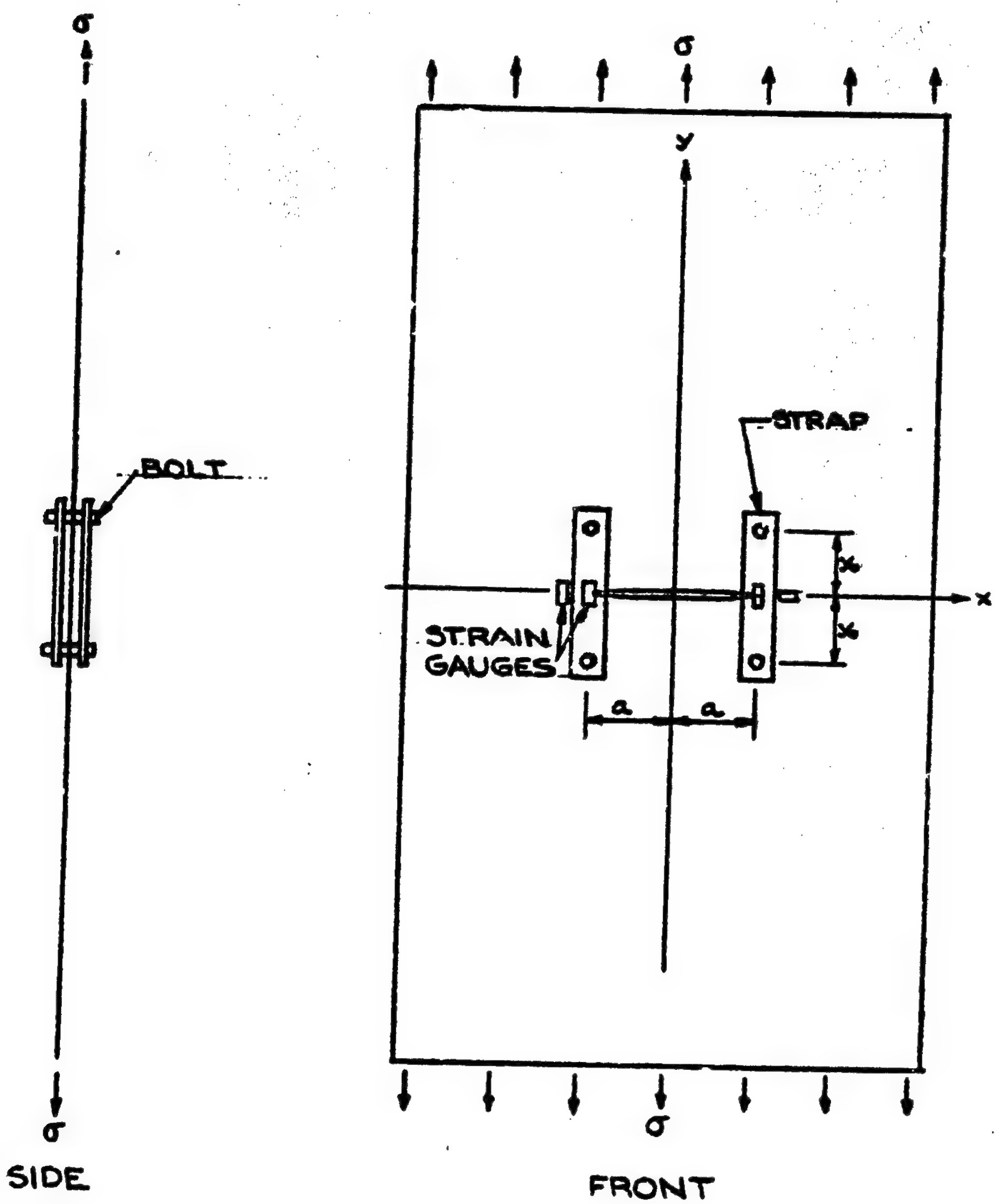


Figure 9 - Specimen Used to Evaluate Effect of Pinching Forces

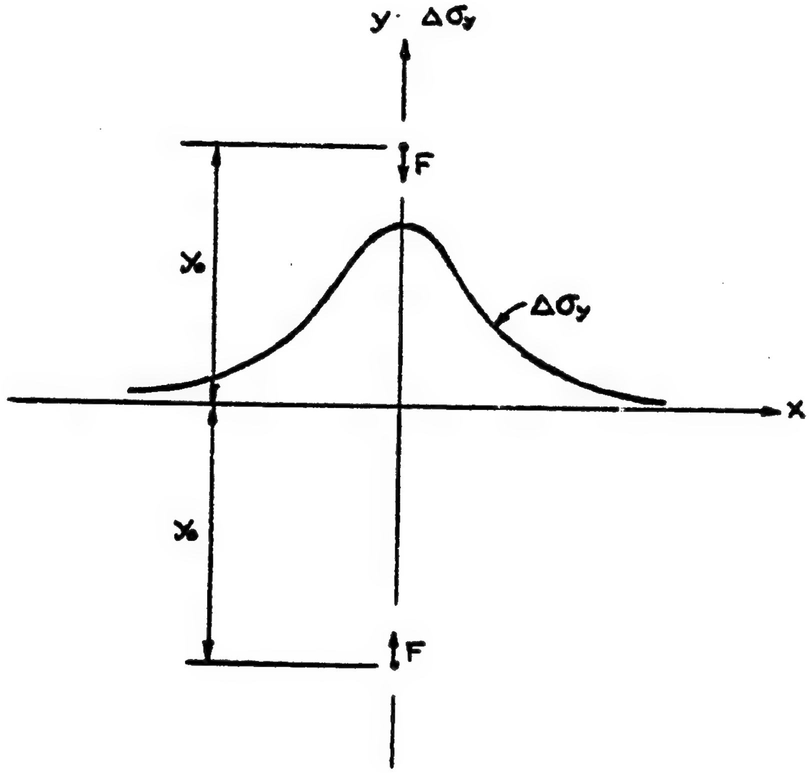


Figure 10 - Pressure Distribution for Pair of Opposed Forces in a Plate

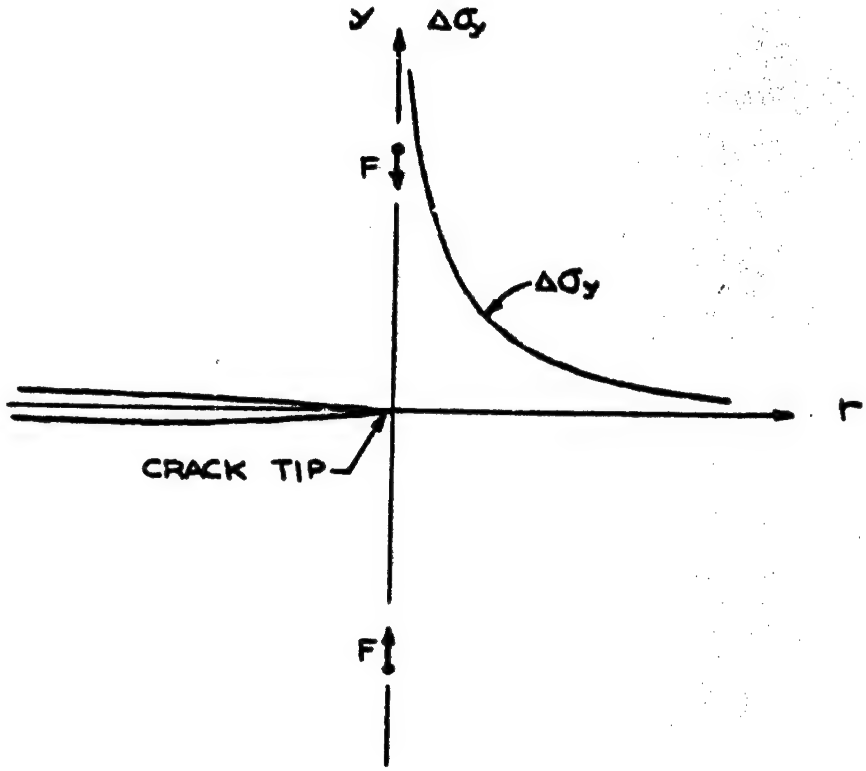


Figure 11 - Additional Pressure Distribution for Pair of Opposed Forces with Crack Present Total distribution = Sum of Figures 10 and 11

TABLE I

TEST I SPECIMEN: ALCLAD 7075-T6; 72" × 24" × 1/16"

FOLLOWING DATA IS THE AVERAGE OF TWO TESTS

$a = 1.0"$ $r = 0.25"$ STRAIN × 10^6

LOAD	REMOTE STRESS	LEFT ϵ_y	LEFT ϵ_x	RIGHT ϵ_y	RIGHT ϵ_x	AVE ϵ_y	AVE ϵ_x
500	333	+ 35.0	+ 2.5	+ 51.5	+ 2.0	+ 43.3	+ 2.3
1000	667	83.0	4.5	98.5	3.0	90.7	3.7
1500	1000	134.0	13.0	144.0	4.5	139.0	8.7
2000	1333	184.0	16.5	190.5	8.0	187.3	12.3
2500	1667	232.0	24.0	235.0	9.5	233.5	16.7
3000	2000	281.0	28.5	285.0	10.5	283.0	19.5

LOAD	REMOTE STRESS	$K = \sigma\sqrt{a}$	$\frac{\epsilon_y + \mu\epsilon_x}{10(1-\mu^2)} = \sigma_y$	$\sqrt{2r}\,\sigma_y = K_a$	$.846 K_a = K$
500	333	333	506	358	303
1000	667	667	1057	748	632
1500	1000	1000	1632	1154	976
2000	1333	1333	2201	1556	1315
2500	1667	1667	2750	1942	1642
3000	2000	2000	3330	2350	1988

FORMULAS USED

$$\sigma_y = \frac{E(\epsilon_y + \mu\epsilon_x)}{1-\mu^2} \qquad K_a = \sqrt{2r}\,\sigma_y$$

$$\frac{K_a}{K} = 1 + \frac{3r}{4a} - \frac{5r^2}{32a^2}$$

$$\frac{K_a}{K} = 1 + \frac{3}{16} - \frac{5}{512} = 1.179$$

$E = 10^7$ psi $\mu = 0.36$

TABLE II

TEST II SPECIMEN: ALCLAD 7075-T6; 72" × 24" × 1/16"

FOLLOWING DATA IS THE AVERAGE OF TWO TESTS

$a = 2.0"$ $r = 0.25"$ STRAIN × 10^6

LOAD	REMOTE STRESS	LEFT ϵ_y	LEFT ϵ_x	RIGHT ϵ_y	RIGHT ϵ_x	AVE ϵ_y	AVE ϵ_x
500	333	+ 52.0	+ 8.5	+ 76.5	+ 14.5	+ 64.3	+ 11.5
1000	667	109.0	23.0	136.5	28.0	122.7	25.5
1500	1000	165.0	38.5	195.5	40.0	180.3	39.3
2000	1333	223.0	51.0	251.5	52.0	237.3	51.5

LOAD	REMOTE STRESS	$K = \sigma\sqrt{a}$	$\frac{E(\epsilon_y + \mu\epsilon_x)}{(1-\mu^2)} = \sigma_y$	$\sqrt{2r}\ \sigma_y = K_a$	$.916\ K_a = K$
500	333	471	785	555	509
1000	667	944	1515	1070	981
1500	1000	1414	2235	1578	1445
2000	1333	1887	2940	2075	1900

FORMULAS USED

$$\sigma_y = \frac{E(\epsilon_y + \mu\epsilon_x)}{1-\mu^2} \qquad K_a = \sqrt{2r}\ \sigma_y$$

$$\frac{K_a}{K} = 1 + \frac{3r}{4a} - \frac{5r^2}{32a^2}$$

$$\frac{K_a}{K} = 1 + \frac{3}{32} - \frac{5}{2048} = 1.092$$

$E = 10^7$ psi $\qquad \mu = 0.36$

TABLE III

TEST III SPECIMEN: ALCLAD 7075-T6; 72" x 24" x 1/16"

FOLLOWING DATA IS THE AVERAGE OF TWO TESTS

$a = 3.0"$ $r = 0.25"$ STRAIN $\times 10^6$

LOAD	REMOTE STRESS	LEFT ϵ_y	LEFT ϵ_x	RIGHT ϵ_y	RIGHT ϵ_x	AVE ϵ_y	AVE ϵ_x
500	333	+ 80.5	+ 28.0	+ 64.5	+ 22.0	+ 72.5	+ 25.0
1000	667	143.0	50.5	132.5	41.5	137.7	46.0
1500	1000	209.0	71.5	200.0	63.0	204.5	67.3
2000	1333	271.0	92.0	269.5	85.5	270.3	88.7

LOAD	REMOTE STRESS	$K = \sigma\sqrt{a}$	$\frac{E(\epsilon_y+\mu\epsilon_x)}{1-\mu^2}$	$\sqrt{2r}\ \sigma_y = K_a$	$.942 K_a = K$
500	333	577	937	662	624
1000	667	1157	1773	1255	1182
1500	1000	1732	2630	1860	1751
2000	1333	2312	3470	2460	2318

FORMULAS USED

$$\sigma_y = \frac{E(\epsilon_y + \mu\epsilon_x)}{1-\mu^2} \qquad K_a = \sqrt{2r}\ \sigma_y$$

$$\frac{K_a}{K} = 1 + \frac{3r}{4a} - \frac{5r^2}{32a^2}$$

$$\frac{K_a}{K} = 1 + \frac{3}{48} - \frac{5}{4610} = 1.061$$

$E = 10^7$ psi $\qquad \mu = 0.36$

TABLE IV

TEST IV SPECIMEN: ALCLAD 7075-T6, 72" × 24" × 1/16"

VOLTAGE APPLIED TO BRIDGE: 17.4

LOAD	MILLIVOLTMETER
0	0.00
1000	0.20
2000	0.42
3000	0.67
4000	0.88
5000	1.13
10000	2.28
15000	3.40
20000	4.58
30000	6.91
Fracture	7.23

FRACTURE LOAD FROM FIGURE 8: 31.5 KIPS

INITIAL CRACK LENGTH, 2a = 6.00"

CRACK LENGTH AT ONSET, 2a = 6.25"

$$K_c = \frac{31{,}500\sqrt{3.13}}{1.5} = 37{,}000 \ \#\text{-in}^{-3/2}$$

$$\mathcal{G}_c = \frac{\pi \sigma^2 a}{E} = \frac{(3.14)(21.0)^2(3.13)}{10.7} = 405 \ \#\text{-in}^{-1}$$

TABLE V

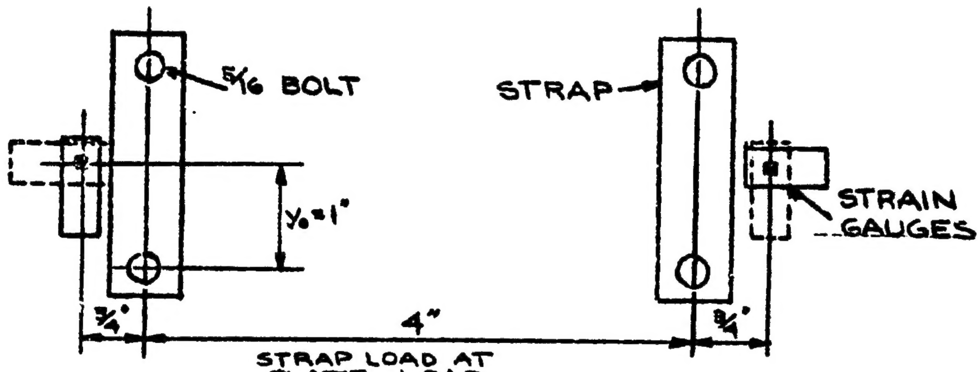
Critical values of the Crack Extension Force and the Stress Intensity Factor for various materials[3]

Material	$\mathcal{G}_c$	K_c
7075-T6 Aluminum (Plane stress; average of about 50 tests at room temperature)	350	34,000
2024-T3 Aluminum (Plane stress; average of about 15 tests at room temperature)	470	39,500
Cast Columbia Resin (CR-39; room temp.)	1	350
British ship steel (Plane strain; room temperature)	153	42,000

TABLE VI

TEST V - TEST OF PLATE WITH STRAPS BUT WITHOUT CRACK TO VERIFY PRESSURE DISTRIBUTION OF PINCHING FORCES

LOADS ARE APPLIED TO PLATE AND RESULTING STRAP LOAD IS DETERMINED THROUGH PREVIOUS CALIBRATION



	STRAP LOAD AT PLATE LOAD			
STRAPS	10,000	20,000	10,000	ZERO
Left	360	535	78	- 320
Right	400	625	175	- 155
	STRAIN $\times 10^6$ AT PLATE LOAD			
	10,000	20,000	10,000	ZERO
ϵ_{y_L}	+ 493	+ 1100	+ 636	+ 128
ϵ_{x_L}	- 181	- 410	- 233	- 40
ϵ_{y_R}	+ 472	+ 1023	+ 575	+ 93
ϵ_{x_R}	- 152	- 356	- 182	- 28
	STRESS AT PLATE LOAD			
	10,000	20,000	10,000	ZERO
σ_{y_L}	4920	10,930	6450	1310
σ_{y_R}	4790	10,280	5850	953

TABLE VI (CONT.)

LOADS APPLIED TO PLATE WITHOUT STRAPS BUT WITH BOLTS IN HOLES

STRAIN × 10^6 AT PLATE LOAD

	10,000	20,000	10,000	ZERO
ϵ_{yL}	+ 628	+ 1258	+ 640	+ 10
ϵ_{xL}	- 183	- 389	- 187	- 11
ϵ_{yR}	+ 622	+ 1257	+ 635	+ 13
ϵ_{xR}	- 253	- 496	- 263	- 10

STRESS AT PLATE LOAD

	10,000	20,000	10,000	ZERO
σ_{yL}	6170	12,420	6380	80
σ_{yR}	6400	12,800	6520	115

STRESS DUE TO STRAPS ALONE

	10,000	20,000	10,000	ZERO
$\Delta\sigma_{yL}$	- 1250	- 1490	+ 70	+ 1230
$\Delta\sigma_{yR}$	- 1610	- 2520	- 670	+ 838

RATIO OF $\Delta\sigma_y/F$

	10,000	20,000
LEFT	.217	.174
RIGHT	.252	.252

THEORETICAL VALUE OF $\Delta\sigma_y/F$ = .243

AVERAGE OF FOUR READINGS = .224, ERROR 8%

TABLE VII

TEST VII - SIMILAR TO TEST V EXCEPT THAT PINCHING FORCES ARE APPLIED TO ENDS OF 6" CRACK

LOADS ARE APPLIED TO PLATE AND RESULTING STRAP LOAD IS DETERMINED THROUGH PREVIOUS CALIBRATION

STRAP LOAD AT PLATE LOAD

STRAPS	5000	10000	5000	ZERO
LEFT	+ 315	+ 400	- 165	- 530
RIGHT	+ 380	+ 480	+ 25	- 300
AVERAGE	+ 342	+ 440	- 70	- 415

STRAIN $\times 10^6$ AT PLATE LOAD

	5000	10000	5000	ZERO
ϵ_{y_L}	+ 311	+ 710	+ 486	+ 188
ϵ_{x_L}	- 30	- 119	- 42	- 14
ϵ_{y_R}	+ 294	+ 707	+ 442	+ 140
ϵ_{x_R}	+ 50	+ 193	+ 63	+ 2

STRESS AT PLATE LOAD

	5000	10000	5000	ZERO
σ_{y_L}	3450	7660	5410	2110
σ_{y_R}	3580	8920	5340	1610
AVERAGE	3510	8290	5380	1860

TABLE VII (CONT.)

LOADS APPLIED TO PLATE WITHOUT STRAPS

STRAIN × 10^6 AT PLATE LOAD

	5000	10000	5000	ZERO
ϵ_{yL}	+ 430	+ 809	+ 419	- 8
ϵ_{xL}	- 13	- 141	- 15	+ 5
ϵ_{yR}	+ 458	+ 968	+ 464	+ 3
ϵ_{xR}	+ 50	+ 237	+ 49	- 2

STRESS AT PLATE LOAD

	5000	10000	5000	ZERO
σ_{yL}	4910	8700	4760	- 70
σ_{yR}	5470	12310	5540	+ 20
AVERAGE	5190	10500	5150	- 50

AVERAGE TAKEN TO REDUCE BUCKLING

STRESS DUE TO STRAPS ALONE

	5000	10000
AVE $\Delta\sigma_y$	- 1660	- 2210
AVE $\Delta\sigma_y/F$	.307	.314

THEORETICAL VALUE OF $\Delta\sigma_y/F = .343$

AVERAGE OF TWO READINGS = .310, ERROR 10%

THIS LARGE ERROR IS ATTRIBUTABLE TO SIGNIFICANT BUCKLING OF SPECIMEN